Physics I
ISI B.Math
Back Paper Exam : January 5, 2011

Total Marks: 100. Time: 3 hours
Answer ALL Questions

Question 1. Total Marks:20

A body of mass m is thrown vertically upwards in air with an initial speed u. In addition to the force of gravity, there is air resistance given by $-mK\mathbf{v}$ where K is a positive constant and $\mathbf{v}$ is the velocity of the body. Find the velocity of the body at all times t and show that its speed cannot exceed a certain value. Find this maximum speed. Also find the time taken to reach the maximum height, and the maximum height reached.

Question 2. Total Marks:7+8+5=20

a.) Show that in an elastic collision of two particles, the speed of each particle in the Center of Mass frame remains same before and after the collision.

b.) If a particle of mass m with speed u, collides elastically with another particle of same mass at rest, show that after the collision either the first particle will come to a complete rest (what will happen to the other particle in this case?), or the particles will bounce off at right angles to each other.

c.) A particle of mass m and speed u collides with another of same mass at rest. After the collision, they stick together. Find the loss of kinetic energy in this collision in terms of m and u.

Question 3. Total Marks:20

A damped harmonic oscillator satisfies the equation

$\ddot{x} + 2K\dot{x} + \Omega^2 = 0$

where K and Ω are positive constants and $K < \Omega$ (under damping). If the particle is released from rest at x=a at t = 0, show that the subsequent motion is given by

$$x = ae^{-Kt}\left(\cos(\Omega_D t) + \tfrac{K}{\Omega_D}\sin(\Omega_D t)\right)$$

where $\Omega_D = (\Omega^2 - K^2)^{1/2}$.

Find all the turning points of the function $x(t)$ and show that the ratio of successive maximum values of x is $e^{-2\pi K/\Omega_D}$.

Question 4. Total Marks:20

Consider two particles with same mass m connected to each other and to two walls by three springs. The three springs have the same spring constant k. Write down the Lagrangian using coordinates x_1 and x_2 relative to the equilibrium position of the particles (ignore gravity). Determine the frequencies of the normal modes of vibration Find the most general solution for the position of the masses as function of time.

Question 5. Total Marks:12+8=20

Show that for any system of particles if the CM is moving at constant velocity $\mathbf{u}$ with respect to another frame of reference S, then

$$K_S = K_{CM} + (1/2)Mu^2$$

where K_S is the kinetic energy in the S frame and K_{CM} is the kinetic energy in the center of mass frame.

Using this result show that if kinetic energy is conserved in one frame then it is conserved in any other frame.